Development of a Node-positioning Algorithm for Wireless Sensor Networks in 3D Space

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***Abstract*— Wireless sensor networks are finding increasing application in various areas of our lives. Typically, the sensor network is important not only to detect or measure the value of an event parameter of interest, but also to correlate it to a specific point in space. As the location of sensors on the ground can be random in nature, there is a question of finding the coordinates of all nodes. Thus, the problem solved in the article has both theoretical and practical value that is extremely important. The algorithm of determining locations of sensor network nodes in three-dimensional space is described in the article.**

***Keywords*— Wireless sensor network, 3D space, node-positioning**

I. Introduction

A wireless sensor network (WSN) is a collection of sensors and actuators communicating with each other through the wireless links. The availability of redundant paths between nodes provides high reliability and self-healing of the network in case of failure.

Today, WSNs are used for a wide range of purposes, such as environment control and monitoring, natural disaster alerting, performing perimeter surveillance.

To accomplish these tasks, the actual node positions should be known. Sometimes, especially in rural and hard-to-reach areas (woodlands, mountain terrain, etc.), the placement of nodes is random rather than perfectly controlled. Therefore, sensor positioning, taking into account the topography of the terrain (i.e., 3-dimensional space), is a crucial issue in WSN operation and management.

There are many investigation papers in the WSN area. The cluster head selection algorithms are the one of the most popular investigation areas on the WSN. The cluster head selection algorithms to the WSN with fixed sensor nodes were developed in the [1,2]. The papers [3,4,5] have extended this results to WSN with mobile sensor nodes. The cluster head selection algorithms to 3D WSN were proposed in the [6,7,8]. The special investigation topic on the WSN with temporary cluster head nodes was considered in the [9]. The FUSN (Flying Ubiquitous Sensor Networks) was investigated in [10, 11]. The localization of sensor nodes in 3D space investigation process is opened now [12], but various research issues are still unsolved in this field.

II. Description of the Subject Area

Let us consider a WSN with an irregular topology consisting of a number of nodes Y_n, $n=1...N$, whose coordinates in 3D space $[x_n \quad y_n \quad z_n]$ are unknown. At the first stage of development of the algorithm, let us assume that there are redundant paths between nodes and all transmissions are performed under the idealized conditions.

In this case, the distance d_{mn} between nodes Y_n and Y_m can be found using the Friis transmission equation:

$$d_{mn} = \frac{\lambda}{4\pi}\sqrt{\frac{P_{t\,m}G_tG_r}{P_{r\,n}}}, \tag{1}$$

where $P_{t\,m}$ is the output power to the transmitting antenna of the m-th node (W);

$P_{r\,n}$ is the power available at the input of the receiving antenna of the n-th node (W);

G_t is the transmitting antenna gain;

G_r is the receiving antenna gain;

λ is the wavelength (m).

The distance d_{mn} can be also obtained as

$$d_{mn} = \left|\overline{Y_mY_n}\right| = \sqrt{(x_m - x_n)^2 + (y_m - y_n)^2 + (z_m - z_n)^2}, \tag{2}$$

where x_m, x_n, y_m, y_n, z_m, z_n are the coordinates of nodes Y_m and Y_n.

In K-dimensional space, the vector of coordinates of any node $\mathbf{Y}_n$ can be defined as a linear combination of K known and linearly independent vectors $\mathbf{Y}_k$, $k=1...K$, $k\neq n$:

$$\mathbf{Y}_n = \sum_{k=1}^{K}\alpha_k\mathbf{Y}_k. \tag{3}$$

Thus, in order to determine the position coordinates of any node, it is enough to know the position coordinates of 3 neighboring nodes, which results in finding coordinates of the tetrahedron vertices.

III. ALGORITHM DEVELOPMENT

At the first stage of determination of node positions, let us assume that there are 3 nodes Y_1, Y_2, Y_3, whose coordinates are known (Figure 1a). These nodes are located in radio range of each other. Node Y_4 with unknown coordinates is also in radio range of nodes Y_1, Y_2, Y_3.

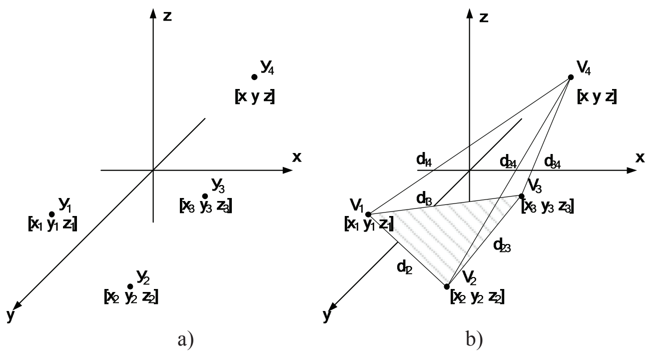

Figure 1. a) node placement in 3D space; b) construction of a tetrahedron with sensor nodes at the vertices

Consider a tetrahedron, which vertices $V_1 \ldots V_4$ have the same coordinates as nodes $Y_1 \ldots Y_4$, respectively. Let us denote the vectors specifying coordinates of the tetrahedron vertices in 3D space as $\mathbf{V}_k$.

The length of tetrahedron edges connecting vertices $V_1 \ldots V_4$, can be found as (2). Taking into account the previously introduced assumptions, we can suppose that the length of the tetrahedron edges between vertices $V_1 \ldots V_4$, computed according to (2), matches the distance d_{mn}, which can be calculated according to (1).

To simplify determination of the coordinates of vertex V_4, let us introduce a new coordinate system using shift (Figure 2a) and rotation (Figure 2b) of the original one:

$$\text{shift } \mathbf{W}_k = \begin{bmatrix} x''_k & y''_k & z''_k \end{bmatrix} = \mathbf{V}_k - \mathbf{V}_1; \qquad (4)$$

$$\text{rotation } \mathbf{y}_k = \begin{bmatrix} U_k & V_k & W_k \end{bmatrix} = \begin{bmatrix} x'_k & y'_k & z'_k \end{bmatrix} = \mathbf{W}_k \mathbf{R} \qquad (5)$$

where $\mathbf{R}$ is a rotation matrix of the coordinate system.

The rotation matrix $\mathbf{R}$ enables to transform vector $\overrightarrow{Y_1Y_2}$ to lie along one of the coordinate axes d_{12}:

transform $\overrightarrow{Y_1Y_2}$ to lie along the X-axis

$$\begin{bmatrix} d_{12} & 0 & 0 \end{bmatrix}: \mathbf{R} = \mathbf{R}_{X1}\mathbf{R}_{Z2};$$

transform $\overrightarrow{Y_1Y_2}$ to lie along the Y-axis

$$\begin{bmatrix} 0 & d_{12} & 0 \end{bmatrix}: \mathbf{R} = \mathbf{R}_{Y1}\mathbf{R}_{X2};$$

transform $\overrightarrow{Y_1Y_2}$ to lie along the Z-axis

$$\begin{bmatrix} 0 & 0 & d_{12} \end{bmatrix}: \mathbf{R} = \mathbf{R}_{Z1}\mathbf{R}_{Y2}.$$

The positions of the other tetrahedron vertices will be defined by coordinates $\begin{bmatrix} U_k & V_k & W_k \end{bmatrix}$.

Here the rotation matrices along with one of the coordinate axes are given as:

$$\mathbf{R}_{Xi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_i & -\sin\alpha_i \\ 0 & \sin\alpha_i & \cos\alpha_i \end{bmatrix} \mathbf{R}_{Yi} = \begin{bmatrix} \cos\beta_i & 0 & \sin\beta_i \\ 0 & 1 & 0 \\ -\sin\beta_i & 0 & \cos\beta_i \end{bmatrix} \mathbf{R}_{Zi} = \begin{bmatrix} \cos\gamma_i & -\sin\gamma_i & 0 \\ \sin\gamma_i & \cos\gamma_i & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where i=1,2.

Eulerian angles α_i, β_i and γ_i are given as:

$$\alpha_i = \arcsin\left(\frac{Z_i}{\sqrt{Y_i^2 + Z_i^2}}\right), \beta_i = \arcsin\left(\frac{X_i}{\sqrt{X_i^2 + Z_i^2}}\right), \gamma_i = \arcsin\left(\frac{Z_i}{\sqrt{Y_i^2 + Z_i^2}}\right),$$

where X_1, Y_1, Z_1 are coordinates x''_2, y''_2, z''_2 of Y_2, obtained as a result of the shift;

X_2, Y_2, Z_2 are coordinates x'_2, y'_2, z'_2 of Y_2, obtained as a result of multiplying $\begin{bmatrix} x''_2 & y''_2 & z''_2 \end{bmatrix}$ by $\mathbf{R}_{X(Y,Z)1}$.

Consider the case of changing coordinate systems with $\mathbf{R} = \mathbf{R}_{Z1}\mathbf{R}_{Y2}$. This transformation results in new coordinates of the tetrahedron vertices: [0, 0, 0] for V_1, $\begin{bmatrix} 0 & 0 & d_{12} \end{bmatrix}$ for V_2, $\begin{bmatrix} U_3 & V_3 & W_3 \end{bmatrix}$ for V_3. The coordinates of vertex V_4 are given as $\begin{bmatrix} U_4 & V_4 & W_4 \end{bmatrix}$. Let us denote $\begin{bmatrix} U_4 & V_4 & W_4 \end{bmatrix}$ as $\begin{bmatrix} A & B & C \end{bmatrix}$.

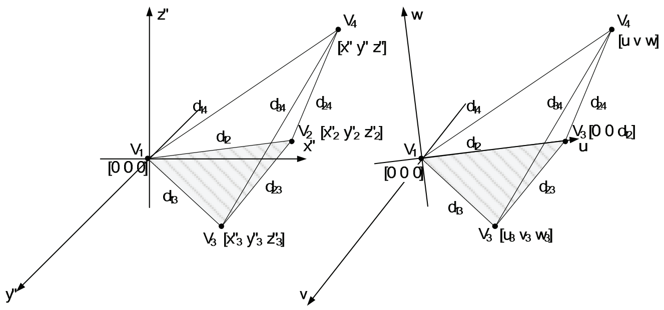

Figure 2. a) shift of the coordinate system; b) rotation of the coordinate system.

Hence, squares of the edge lengths between vertices $Y_1 \ldots Y_3$ and vertex Y_4 are defined by the system of equations:

$$\begin{cases} d_{14}^2 = A^2 + B^2 + C^2 \\ d_{24}^2 = A^2 + B^2 + (C - d_{12})^2 \\ d_{34}^2 = (A - U_3)^2 + (B - V_3)^2 + (C - W_3)^2 \end{cases} \qquad (6)$$

Let us find the difference of squares between the edge lengths of the tetrahedron:

$$\begin{cases} d_{14}^2 - d_{24}^2 = 2 \cdot C \cdot d_{12} - d_{12}^2 \\ d_{24}^2 - d_{34}^2 = 2AU_3 - U_3^2 + 2BV_3 - V_3^2 + d_{12}^2 - 2Cd_{12} + 2CW_3 - W_3^2 \\ d_{34}^2 - d_{14}^2 = (A - U_3)^2 + (B - V_3)^2 + (C - W_3)^2 - A^2 - B^2 - C^2 \end{cases} \qquad (7)$$

Taking into account that the coordinates of vertices $Y_1 \ldots Y_3$ are known, we can uniquely determine one of the coordinates of vertex Y_4 (in this case, it is C):

$$C = \frac{d_{12}^2 + d_{14}^2 - d_{24}^2}{2d_{12}} = W_4, \qquad (8)$$

Substituting the obtained coordinate W_4 into either one of the remaining equations, $d_{24}^2 - d_{34}^2$ or $d_{34}^2 - d_{14}^2$, enables to define the relation between coordinates A and B: $B=kA+l$, where

$$k = \frac{U_3}{V_3}; \quad l = \frac{U_3^2 + V_3^2 + d_{14}^2 - d_{34}^2 + (W_4 - W_3)^2 - W_4^2}{2V_3} \quad (9)$$

Substitution of B and C in equation $d_{14}^2 - d_{24}^2$ yields $d_{14}^2 = aA^2 + bA + c$, the roots of which correspond to 2 possible coordinates of Y_4 along the u-axis (U_{41} and U_{42}) and, consequently, 2 possible coordinates along the v-axis (V_{41} and V_{42}).

In order to return to the original coordinate system, one should perform the inverse transformations:

$$\mathbf{Y}_k = \mathbf{V}_k \mathbf{L} + \mathbf{V}_1, \quad (10)$$

where $\mathbf{L} = \mathbf{R}_{Z2}^T \mathbf{R}_{X1}^T$ for the X-axis; $\mathbf{L} = \mathbf{R}_{X2}^T \mathbf{R}_{Y1}^T$ for the Y-axis; $\mathbf{R} = \mathbf{R}_{Y2}^T \mathbf{R}_{Z1}^T$ for the Z-axis.

An obvious drawback of the developed node-positioning algorithm is the need to solve the quadratic equation, which leads to 2 possible coordinates, $[x_{41} \quad y_{41} \quad z_{41}]$ and $[x_{42} \quad y_{42} \quad z_{42}]$, and makes impossible the localization of next nodes in the network, since the number of possible positions in 3D space grows exponentially with the number of nodes, resulting in 1024 possible locations for the 10th node.

This problem can be solved by using one more node, Y_5, with known coordinates and interacting with nodes $Y_1 \ldots Y_4$.

Substitution of one of nodes $Y_1 \ldots Y_3$ by node Y_5 as a tetrahedron vertex leads to solutions $[x_{43} \quad y_{43} \quad z_{43}]$, $[x_{44} \quad y_{44} \quad z_{44}]$.

Since one of the solutions is $[x_{43} \quad y_{43} \quad z_{43}] = [x_{41} \quad y_{41} \quad z_{41}]$, this gives us the actual position of Y_4 in 3D space. With the ability to build 3 tetrahedrons with bases V_1 V_3 V_5, V_1V_2 V_5, V_2 V_3 V_5, we can determine the coordinates of node V_4, even in the presence of interference, introducing errors in the calculation of the distances between nodes. Positioning of the rest of the nodes in the network is performed according to the algorithm described above.

IV. Future Work

This paper attempts to determine the position of the nodes in the three-dimensional space, based on the presence of redundant links between network nodes according to the signal level.

The described above algorithm provides the ideal conditions for the transfer, but in practice this is not true.

Arriving at the receiver input signal undergoes multipath propagation, fading in the wireless channel it depends on many factors, in particular weather conditions.

Given that the channel parameters between any network nodes are not known (because they are unknown position) and the route between them, therefore, the algorithm needs to be modified using the adaptive signal processing techniques. We should also consider the case where all the check nodes are in the same plane, which results in two positions with respect to the ground node.

V. Conclusions

This line of research is relevant because wireless sensor networks have a number of advantages over previous-generation communication networks, the most important of which are the self-organization and the possibility of their use for the data transmission in places where the use of the wired networks is not possible.

FSU models that are discussed in various studies are often applied in nature and, accordingly, only some of the factors that influence those or other characteristics of a single network model are taken into account.

All these facts emphasize the necessity of development of the new algorithms of sensor nodes position definition considering as the saved-up experience, and perhaps new parameters once again.

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